

Number system

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Contains:

- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

Decimal number system

- 0,1,2,3,4,5,6,7,8,9 - ten unique symbol, each symbol is called 'digit'.
- Hence called base or radix is 10
- Positional weighted system means value attached with symbol depends upon its position where it placed with respect to decimal point.
- MSB – Most Significant Bit – greatest positional weight out of all number
- LSB – Least Significant Bit – lowest positional weight out of all number

➤ 9's complement

- **Need of complement in digital number system** :Complements are used by a machine to represent negative number. By taking the complement of a negative number, the number can be converted into another number suitable for machines.
- 9's complement can be perform by subtracting each digit from 9
- If carry is generated answer is called positive , add it to least bit is called all around carry
- If no carry is generated answer is called negative
- Example : 3652 , 695.65

9999

-3652

6347

9's complement

999.99

-695.65

304.34

➤ 10's complement:

- Is generated by adding 1 to 9's complement.
- Example : 3652 , 695.65

$$\begin{array}{r} 9999 \\ -3652 \\ \hline 6347 \quad \text{9's complement} \\ + \quad 1 \\ \hline 6348 \quad \text{10's complement} \end{array}$$

$$\begin{array}{r} 999.99 \\ -695.65 \\ \hline 304.34 \\ + \quad 1 \\ \hline 304.35 \end{array}$$

➤ Subtraction using 9's and 10's complement

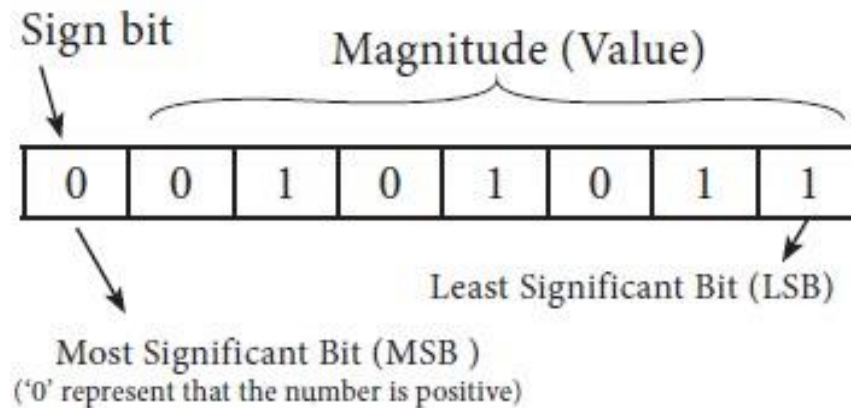
1. $896.66 - 659.15 = 421.30$

9's complement	steps
896.66	
+ 524.63	9's complement of number to be subtracted
①421.29	Result
1	Add around carry
421.30	Answer

10's complement	steps
896.66	
+ 524.64	10's complement of number to be subtracted
① 421.30	Result
	Ignore carry
421.30	Answer

▪ Signed and Unsigned number:

- In sign magnitude form, extra bit is added is known as sign bit.
- If sign bit is 1- number is negative
- If sign bit is 0 – number is positive



Binary number system:

- Only two digits 0 and 1 and base or radix is 2
- Numbers of column is decided by 2^n
i.e for $2^1 = 2$ so 0,1
 $2^2 = 4$ so 00,01,10,11 further on.....
- Binary number with its decimal value is shown in below table:

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110

➤ Binary to decimal conversation:

Convert number $(100101.11)_2$ to decimal

- Method – 1:

positional weights – $2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \ 2^{-1} \ 2^{-2}$

1 0 0 1 0 1 . 1 1

$$= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 32 + 0 + 0 + 4 + 0 + 1 + 0.5 + 0.25$$

$$= 37.75$$

- Method – 2:

MSB = 1

Multiply by 2, add next bit $(1 \times 2) + 0 = 2$

Multiply by 2, add next bit $(2 \times 2) + 0 = 4$

Multiply by 2, add next bit $(4 \times 2) + 1 = 9$

Multiply by 2, add next bit $(9 \times 2) + 0 = 18$

Multiply by 2, add next bit $(18 \times 2) + 1 = 37$

➤ Decimal to Binary conversation:

Convert Number 183.875

- **Method – 1: Sum of weights**

Subtract number from its maximum power 2 number

As $2^7 = 128$ and $2^8 = 256$, so $183 - 128 = 55$ and $2^7 = 1000000$

Now same way $2^5 = 32$ so $55 - 32 = 23$ and $2^5 = 10000$

$2^4 = 16$ so $23 - 16 = 7$ and $2^4 = 1000$

$2^2 = 4$ so $7 - 4 = 3$ and $2^2 = 10$

$2^1 = 2$ so $3 - 2 = 1$ and $2^1 = 1$

Now reminder is only 1 which is power of 2^0 so it will be as it is = 1

Now add them all $1000000 + 10000 + 1000 + 10 + 1 + 1 = 10110111$ (Binary addition)

For Fractional part process will be same but it will be in 2^{-1} format

So for $875 = (111)_2$

Continue...

Convert Number 183.875

- **Method – 2: Double Dabble**

Successive	Division	Reminder
2	183	
2	91	1
2	45	1
2	22	1
2	11	0
2	5	1
2	4	1
2	2	0
2	1	1
	0	

Fractional	Answer	Reminder
Multiply 0.875 by 2	1.750	1
Multiply 0.750 by 2	1.5	1
Multiply 0.5 by 2	1.0	1

So Answer will be $(183.875)_{10} = (10110111.111)_2$

➤ Binary Addition, subtraction, multiplication, division:

$$\begin{array}{r}
 \overset{11}{1011.01} \\
 + \quad \overset{1}{11.011} \\
 \hline
 1110.101
 \end{array}$$

$$\begin{array}{r}
 1011.01 \\
 \times \quad 110.1 \\
 \hline
 101101 \\
 000000 \\
 10110100 \\
 101101000 \\
 \hline
 1001001.001
 \end{array}$$

$$\begin{array}{r}
 11.11\overline{10} \\
 11 \overline{)1011.11000} \\
 \underline{11} \downarrow \\
 101 \downarrow \\
 \underline{11} \downarrow \\
 101 \downarrow \\
 \underline{11} \downarrow \\
 101 \downarrow \\
 \underline{11} \downarrow \\
 100 \downarrow \\
 \underline{11} \downarrow \\
 100 \downarrow \\
 \dots \\
 \dots
 \end{array}$$

Binary Subtraction	Decimal Subtraction
$ \begin{array}{r} 010 \\ 10101 \\ - 10010 \\ \hline 00011 \end{array} $	$ \begin{array}{r} 21 \\ - 18 \\ \hline 3 \end{array} $
$00011_2 = 3_{10}$	

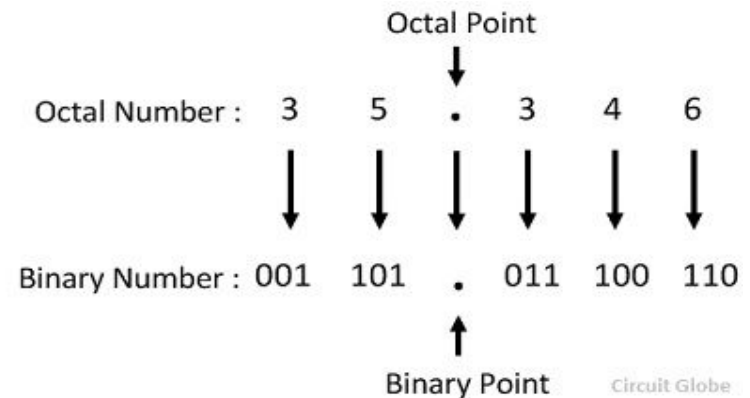
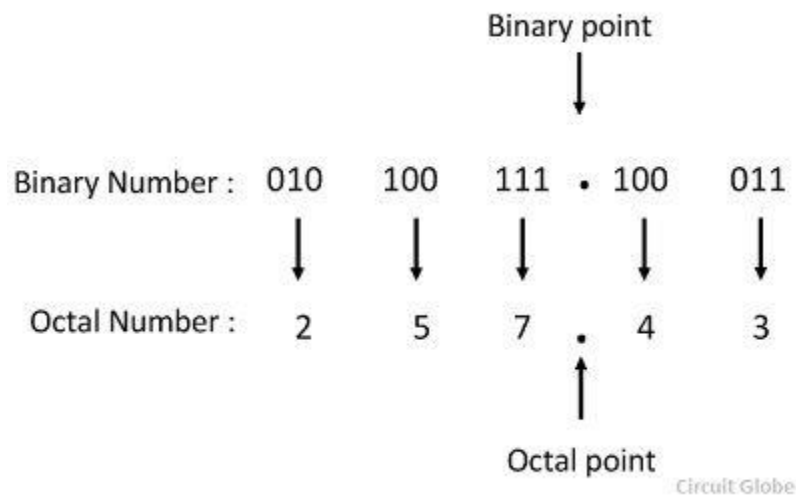
Octal Number system:

- Radix or base is 8
- 8 independent symbol – 0,1,2,3,4,5,6,7
- More useful than binary while dealing with large number of data

Decimal	Octal	Binary
0	0	0
1	1	1
2	2	10
3	3	11
4	4	100
5	5	101
6	6	110
7	7	111

➤ Octal to binary and vice versa:

- Its very simple to convert number to binary
- For any binary to octal conversation just make group of 3 i.e. for any binary number from its right most starts making group of 3 and put its octal value.
- For any octal to binary conversation, any octal digit put its 3 digit binary value.



➤ Octal to Decimal and vice versa:

- The process will be same as was in binary to decimal conversation

3	7	2	4	6
---	---	---	---	---

8^4	8^3	8^2	8^1	8^0	
└───┬───┬───┬───┬───┘	└───┬───┬───┬───┘	└───┬───┬───┘	└───┬───┘	└───┘	→
					$6 \times 8^0 = 6$ $4 \times 8^1 = 32$ $2 \times 8^2 = 128$ $7 \times 8^3 = 3584$ $3 \times 8^4 = 12288$ <hr style="width: 100%;"/> 16038

Octal to Decimal conversion

Octal = **37246**

Decimal = **16038**

8	100	4	↑	144
8	12	4	↑	
8	1	1	↑	

(100)₁₀ = (144)₈

Decimal to Octal Conversation

Read in reverse order

➤ Octal Addition, Subtraction, Multiplication, Division:

$$\begin{array}{r}
 11 \quad \text{carry} \\
 456 = 302_{10} \\
 +123 = 83_{10} \\
 \hline
 601 = 385_{10}
 \end{array}$$

$$\begin{array}{r}
 \text{Octal} \\
 762 \\
 \times 45 \\
 \hline
 4672 \\
 3710 \\
 \hline
 43772
 \end{array}$$

<u>Octal</u>	=	() ₁₀	=	(8 ×) +	=	<u>Octal</u>
5 × 2	=	(10) ₁₀	=	(8 × 1) + 2 =		12
5 × 6 + 1	=	(31) ₁₀	=	(8 × 3) + 7 =		37
5 × 7 + 3	=	(38) ₁₀	=	(8 × 4) + 6 =		46
4 × 2	=	(8) ₁₀	=	(8 × 1) + 0 =		10
4 × 6 + 1	=	(25) ₁₀	=	(8 × 3) + 1 =		31
4 × 7 + 3	=	(31) ₁₀	=	(8 × 3) + 7 =		37

Carry

$$\begin{array}{r}
 8 \quad \text{borrow} \\
 {}^3 456 = 302_{10} \\
 -173 = 123_{10} \\
 \hline
 263 = 179_{10}
 \end{array}$$

$$\begin{array}{r}
 156 \overline{) 372.00} \\
 \underline{334} \\
 360 \\
 \underline{334} \\
 240 \\
 \underline{156} \\
 62
 \end{array}$$

Remainder 62

Hexadecimal number system:

- As name implies it has base or radix of 16 and symbols are: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Its machine language
- Group of 4 bit is called nibble and 2 nibble or 8 bit is called word.
- Also known as Hex number system

Hex	Dec	Oct	Bin
00	00	00	00000
01	01	01	00001
02	02	02	00010
03	03	03	00011
04	04	04	00100
05	05	05	00101
06	06	06	00110
07	07	07	00111
08	08	10	01000
09	09	11	01001
0A	10	12	01010
0B	11	13	01011
0C	12	14	01100
0D	13	15	01101
0E	14	16	01110
0F	15	17	01111
10	16	20	10000

➤ Hex to binary and vice versa:

- Its very simple as in octal to binary conversation
- Like in octal, make group of for to convert binary to hex and put 4 digit binary value for any hex digit to convert it to binary

Hexadecimal number :	9	A	F
	↓	↓	↓
Binary Number :	1001	1010	1111

Hex to binary conversation

1111 0001 0000 0010
↓ ↓ ↓ ↓
F102

Binary to Hex Conversation

➤ Hex to Octal and vice versa:

- To convert to octal number , first convert it to binary and then to octal and vice versa.

Hex to octal Conversation

Step 1:

Hex to Binary Conversion

B	5	A
1011	0101	1010

So the binary equivalent is 101101011010

Step 2:

Binary to Octal Conversion

<u>101</u>	<u>101</u>	<u>011</u>	<u>010</u>
5	5	3	2

Octal to Hex Conversation

Step 1:

Octal to Binary Conversion

7	5	2
111	101	010

So the binary equivalent is 111101010

Step 2:

Binary to Hex Conversion

<u>0001</u>	<u>1110</u>	<u>1010</u>
1	D	9

Hex to Decimal and vice versa:

- Method will be same as in binary to decimal conversation. Here we are using base of 16 instead of 2.

$1F4_{16}$

positional powers of 16:	16^3	16^2	16^1	16^0
decimal positional value:	4096	256	16	1
Hexadecimal number:		1	F	4

$$\begin{aligned} & (1 \times 256) + (F \times 16) + (4 \times 1) \\ & = (1 \times 256) + (15 \times 16) + (4 \times 1) \\ & = 256 + 240 + 4 = 500_{10} \end{aligned}$$

16	427	Remainders in hexadecimal
	26	11=B
	1	10=A
	0	1

➤ Hex Addition, subtraction, multiplication, division:

$$\begin{array}{r}
 1 \quad \text{carry} \\
 4A6 = 1190_{10} \\
 + 1B3 = 435_{10} \\
 \hline
 659 = 1625_{10}
 \end{array}$$

$$\begin{array}{r}
 10_{16} \quad \text{Borrow} \\
 2 \quad \text{Minuend reduced by 1} \\
 4A\cancel{7}_{16} \quad \text{Minuend} \\
 - 2C4B_{16} \quad \text{Subtrahend} \\
 \hline
 C_{16} \quad \text{Difference}
 \end{array}$$

$$\begin{array}{r}
 3 4 F \\
 X D E \\
 \hline
 \mathbf{1} \\
 9 C E 5 2 \\
 9 1 B 0 3 \\
 \hline
 9 B 7 E 8 2
 \end{array}
 \quad \left. \vphantom{\begin{array}{r} 9 C E 5 2 \\ 9 1 B 0 3 \\ 9 B 7 E 8 2 \end{array}} \right\} \text{Add up}$$

EX:

$$\begin{array}{r}
 DEF \\
 12\sqrt{FACE} \\
 \underline{EA} \\
 10C \\
 \underline{FC} \\
 10E \\
 \underline{10E} \\
 0
 \end{array}
 \quad \begin{array}{l}
 12 \times D = EA \\
 FA - EA = 10 \\
 12 \times E = 10C \\
 10C - FC = 10 \\
 12 \times F = 10E
 \end{array}$$